The Importance of "Musical" Fractions in Ancient Egypt
By George Essence

The Ancient Egyptians used "fractions" for every mathematical purpose including counting. This conclusion is based entirely on existing data, freely available on the internet, such as the sample that I summarize below.

1)   A quote from Wikipedia.

"An interesting feature of Ancient Egyptian mathematics is the use of unit fractions. The Egyptians used some special notation for 1/2, 1/3, 2/3 and 3/4.Other fractions were all written as unit fractions of the form 1/n or sums of such unit fractions"

As it happens, 1/2, 1/3, 2/3 and 3/4 are the basic intervals of music.

2)   The pyramids made of stone, including the three at Giza, were built between 3100BC and 2500BC. Their slopes are these basic intervals of music (data from a senior surveyor for the Cairo Museum, Mark Lehner).

3)   The stone temple between the paws of the Sphinx has internal dimensions making the same "music" fractions (fractions also found in Chartres cathedral.)

4)   The Sumerian poem "The Epic of Gilgamesh" describes precisely how the fraction 2/3 was used. Gilgamesh was "two-thirds god", and used 2/3 for travel in the stars.

5)   The base of the Great Pyramid is an exact square, covering 13 acres. It is <u>*flat within one inch*</u>. Marked out in a grid of identical squares like modern graph paper, it would have been ideal for geometry, which is based on measurement and angle. Building measurements could be taken directly from drawings on this grid.

"Pythagorean tuning is a system of musical tuning in which the frequency relationships of all intervals are based on the ratio 2/3". (Wikepedia)

My conclusions from this investigation are:

The religion of the Egyptians was based on the Sun as a creator god. He created a goddess, the Moon. Then between them they created the stars as lesser gods.
With these stars they created the 12 zodiac "houses".

By "living in each zodiac house" in turn the Sun was able to give life to the earth, which was a garden belonging to the goddess.

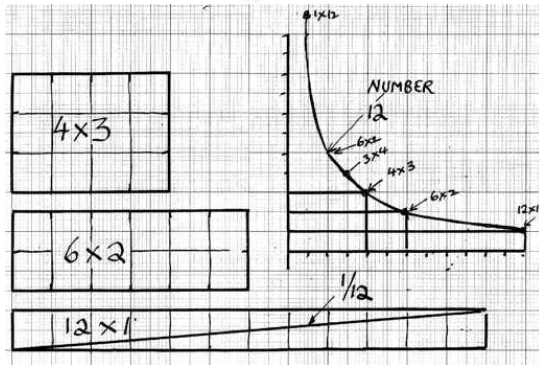The Sun Moon and stars moved using the laws of music, which are the sacred ratios 1/1, 1/2, 2/3 and 3/4.

A fraction was thought of as a rectangle drawn on a grid. Here is an example, below.

RECTANGLE AREA 5

EGYPTIAN FRACTION 1/5

SLOPE OF DIAGONAL 1/5
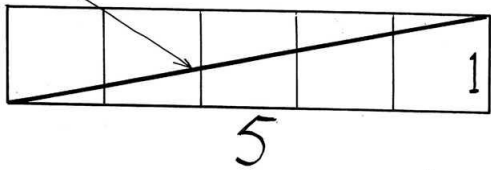
Fractions with 1 on top, like 1/5, were rectangles with a diagonal slope (eg "1/5").

The actual _number_, 5, was the _area_.
It was the number of squares

Every possible rectangle of area 12 say makes a curve on the grid. So every "number" was both an area and a curve. Every possible point in between those shown is a possible rectangle.

Annual floods of the Nile brought down fertile soil every year. Rectangular fields could be redrawn after the flood, keeping the same area, but using the curve to find the most convenient rectangle shape.
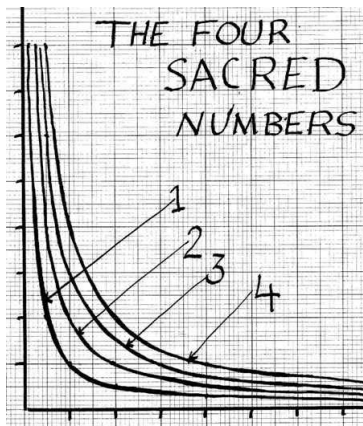
The graphs of the four sacred numbers, 1 2 3 4, are equally spaced. The graph for "1" is the equation
$$XY=1, \text{ or } Y = 1/X$$
If X = 173 then you can measure the tiny distance 1/173 directly off the graph. You build any small amount by adding fractions, (like "shims" in modern engineering)

For example on the famous Rhind papyrus, the numbers 13, 17 and 173 are written. If you make these into fractions, you get a very accurate "Pi".

$$\Pi = 3 +1/13 +1/17 +1/173 = 3.1415$$

The Egyptians used prime numbers for these measurement fractions (to distinguish them from sacred numbers)

It is difficult to get Pi as accurately as this if you didn't know these particular "prime" fractions.
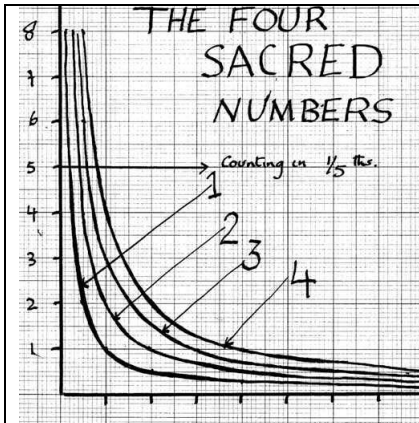
The curves are equal distance apart on every horizontal line.

So the curves make different "scales" to suit every purpose.

These 4 "sacred" curves were carved into cylinder seals

On this Mesopotamian cylinder seal (2300BC) you can see 4 chevrons in the frame.

A rectangle is shown beneath them.

Above them, in contrast, are a pile of rectangles for the sacred fraction "2/3"

Graphs drawn on the grid could be used to calculate and actually measure any dimension necessary for architecture and astronomy. "Mathematical" exactness was not an issue for Ancient Egypt. It is a completely abstract concept that does not occur in nature.

Musical tones on the other hand, which do occur in nature, are not exact. They exist within the ability of the human ear to distinguish pitch, the sort of tolerance also found in practical architecture, building and astronomy.

A modern engineer might give a measurement in a form like "5cm +/- 0.01mm" but sacred fractions are about sound. They are the well known musical intervals between two notes, and they create the 12 equally spaced notes on the circle below.

The circle, called the ecliptic, is the path of the sun. The sun as it moves across the sky is overtaken gradually by 12 groups of stars. These 12 groups of stars are the "zodiac houses". Overtaking the sun ensures that it enters a different zodiac house every month (and a new musical note.)Sacred fractions being musical intervals are also time distances between zodiac houses.
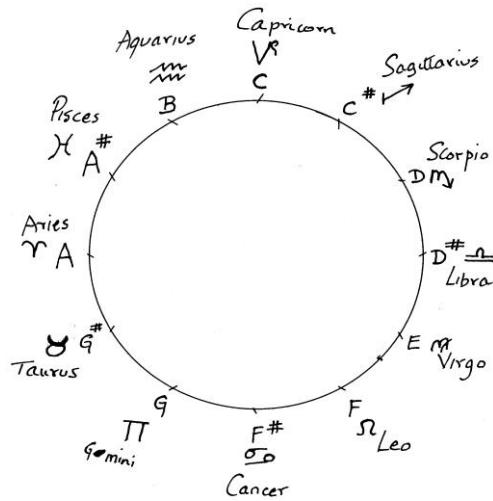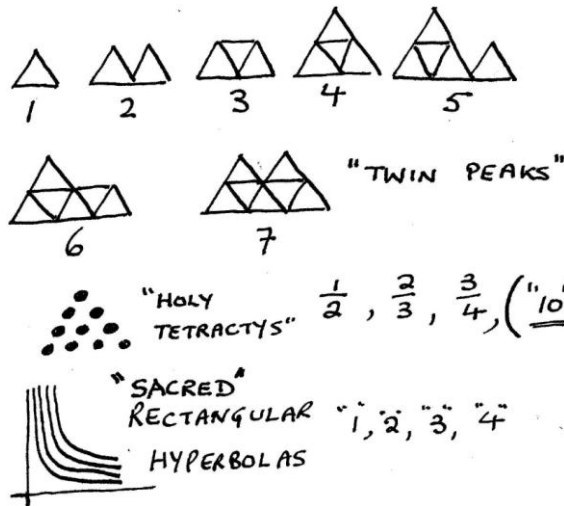
"2/3" is the "interval" used by gods, cosmic beings.

2/3 is a _clockwise_ journey of 7 spaces. It is one step taken by "7 league boots".

F-C clockwise would be one 2/3 step, Leo to Capricorn "non stop". It is a straight line directly across the circle.

"Sacred counting" used a different set of symbols to engineers. Triangles were used to count intervals around the zodiac.

Triangles show steps around the circle of different lengths of Time.

One step is one zodiac age, or 2160 years.

So 2/3, 7 steps, is a period of 15,120 years.

The "holy tetractys" of Pythagoras is the Greek version of this system. The "10" is about Man, Adam. The religion of Egypt is about gods and time. Gods live in heaven and do not age, unless they come to earth, like Adam, when Time causes them problems.
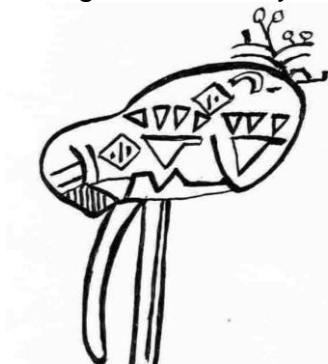
In this sketch from the famous Egyptian Narmer Plate, 3000 BC, the small triangles suggest 3/4.

The bigger triangles could represent time intervals. The overall shape may be a liver used for divination.

4

SUMMARY

1) NUMBERS

Any number can be made into a curve on the grid, but I don't think every number was drawn as a curve. Field sizes could probably be estimated from the curves of the 4 sacred numbers.

Counting numbers were rectangles of the form 1xN, (rectangles of unit height.) Some numbers, 6 say, could be different rectangles, like 1x6 and 2x3, but Prime numbers can only be 1xN, like 1x5. They are "inert" or "lifeless".

Primes were "given life" by multiplication by 2 and 3. Since these belong to the Sun, it is the Sun that gives life to the body as the inert form. The body is inert, and is shown without a head. It lacks "life".

Prime numbers were multiplied by 6, and then by more "2"s if necessary, like 12, 24, 48 etc.Primes all come from the number 6. They are all of the form 6n +1 or 6n-1. For example 5 is 6-1, 37 is 6x6+1, and 173 is 6x29-1. The Primes are given by the sun, too. When Adam as a god came to earth he was given 5 for his body. "5" was also included in the music of the sacred numbers.

They became 1/1, 2/1, 3/2, 4/3, 5/4, 6/5, 9/8, 10/9 and 16/15. 8/5 and 5/3 also appear, as combination fractions .

"Music"

The steps around the circle are

Ratio	1/1	2/1	3/2	4/3	5/4	6/5	9/8	10/9	8/5	5/3
steps	0	12	7	5	4	3	2	2	8	9
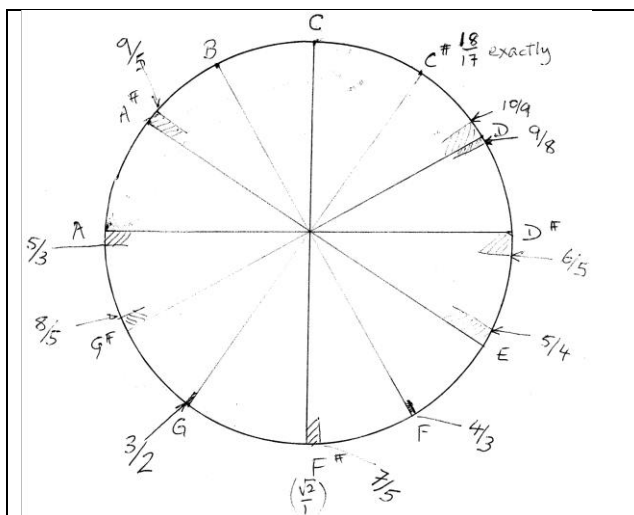
But these new steps are only approximate. The trouble with using 5 is that the new steps are not perfect. Only 1/1, 2/1, 3/2, 4/3 are perfect.

The new fractions make discords, which is why Man was said to be "noisy".

2) PYRAMIDS

The 4th Dynasty pyramids represent sacred numbers with their slopes. This table from the website of Professor Greenberg shows data taken from Mark Lehner, senior surveyor for the Cairo museum.

PYRAMID	ANGLE OF FACE	INVERSE SLOPE OF FACE	ANGLE OF EDGE	INVERSE SLOPE OF EDGE
BENT PYRAMID	54°27'44" 43°22'00"	.714288 1.058703	44°42'36" 33°44'20"	**1/1** **3/2**
RED PYRAMID	43°22'00"	1.058703	33°44'20"	**3/2**
KHUFU ('Great')	51°50'40"	.785667	41°59'15"	**10/9**
KHAFRE	53°10'00"	**4/3**	43°21'07"	1.059250
MENKAURE	51°49'38" 51°10'46"	.786154 .804615	41°38'08"	**9/8**

Where the decimal figures coincide with a sacred fraction, I have substituted the fraction. The only fraction with a "5" in it is the slope of the Great Pyramid. The slopes have meanings within the myth of the religion.

3/2 is the descent of the god, 1/1 is the sun god "resting". The red pyramid is red for Adam because he represents the planet Mars. Edge slopes are for heaven. Face slopes, and the fraction 4/3, are for the Underworld.Face and edge slopes differ by a factor √2. This factor is the musical "tritone" interval, the number "666", and "The Gateway" into the Underworld.

Menkaure is unique because it's base is a rectangle not a square. This means that its edge slopes are all equal but there are two different face slopes. One of them, 51°10'46" , is the latitude of Stonehenge. The angle at its apex is the relative longitude of Stonehenge from Giza (not so close though). The slope height is almost the diameter of the Aubrey holes circle at Stonehenge. Its sacred fraction 9/8 is a Time period.

The Egyptian religion is a myth about the creation of Man. It concerns a Time span of only about 13,000 years, and Man is a god who will live on earth for this period. He has lived here before in three previous "worlds", a point of view common among Native Americans and other "primitive people" like the Hopi.

Apart from the sacred fractions linking England to Egypt in 3000 BC, Rendell Harris among others has commented on Egyptian place names in England.

Stonehenge has dressed stones, a skill not in evidence anywhere else in Britain but was an important skill in Egypt at the time Stonehenge was built.

The shape of the Scottish broch and of "beaker" pottery is the shape of the horns of the constellation of Taurus. The period of the zodiac Age of Taurus is approximately 4600 to 2400 BC

Stonehenge is designed in concentric circles
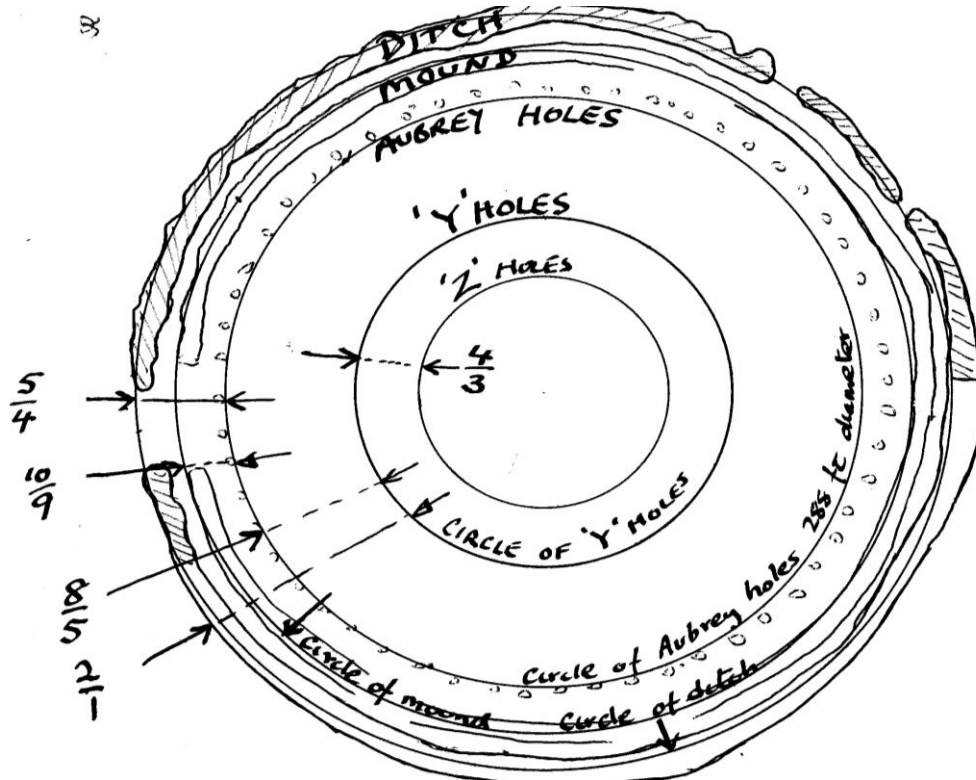
The layout at Stonehenge is in fact 9 concentric circles. For simplicity I show only 5 of them above.
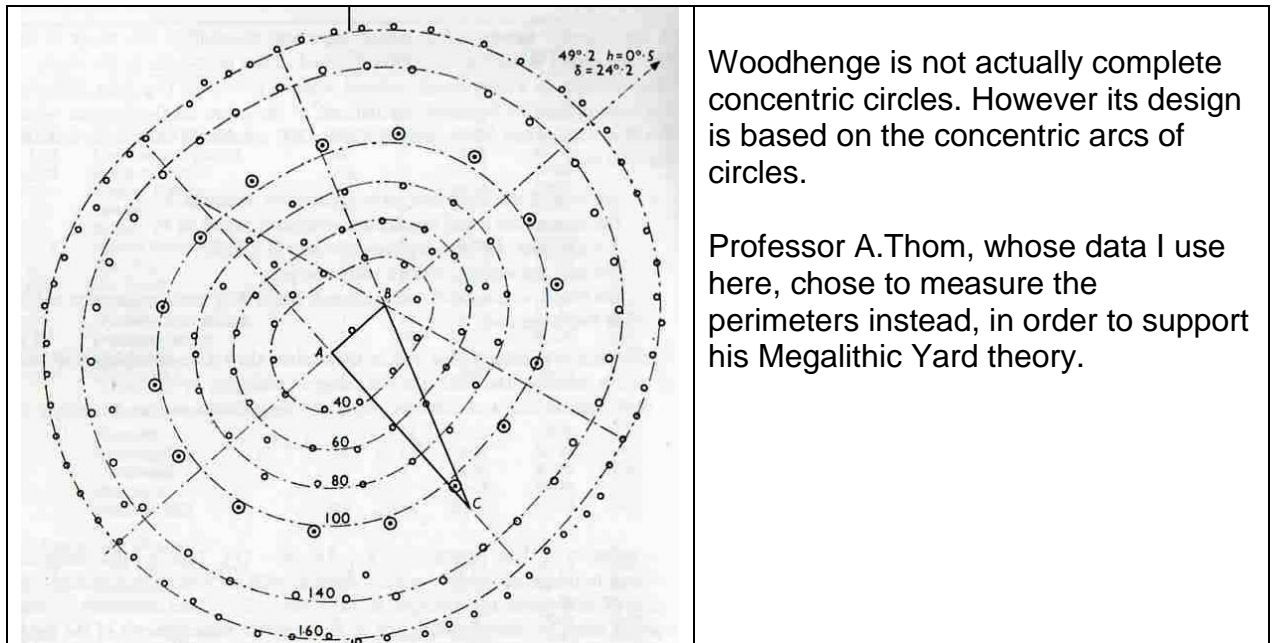
The other circles of the Sarsens, Blue Stones, Q holes and R holes are not shown here. However all 9 diameters are included in the table of ratios shown below.

Ditch/Bank	10/9	Harmonic ratios are shown in the right hand column.	
Bank/Aubrey Holes	9/8		
Aubrey / Y Holes	8/5		
Y holes/ Z holes	4/3	The left hand column gives the two circles whose diameters make the harmonic ratios.	
Z holes/ Sarsen circle	5/4		
Sarsen circle / Blue stone circle	5/4		
Q holes/ R holes	9/8	For example the diameter of the ditch circle divided by the diameter of the Bank circle is 10/9	
Z holes/ Q holes	3/2		
Y holes/ Q holes	2/1		

Every diameter makes at least one harmonic ratio with another. It is not a question of selecting particular diameters. Harmonic ratios appear to be the design criterion for the plan of Stonehenge in all three stages of its construction.

For example two small ditch circles around the north and south barrows (not shown) have diameters 60 and 40 feet, another 3/2 ratio. The 4 Station Stones divide the 56 Aubrey holes in the ratio 1/3.

Woodhenge (Source: "Lunar Observatories" by A.Thom)

Woodhenge is not actually complete concentric circles. However its design is based on the concentric arcs of circles.

Professor A.Thom, whose data I use here, chose to measure the perimeters instead, in order to support his Megalithic Yard theory.

Woodhenge perimeter ratios
Dimensions in Megalithic Yards (MY)

Ring	Perimeter MY actual	Ratio	Ratio value	Closest ratio	fraction
a	161.o	a/b	1.16	1.125	9/8
b	138.2	b/c	1.33	1.33	4/3
c	104.2	c/d	1.31	1.33	4/3
d	79.9	d/e	1.29		4/3
e	61.3	e/f	1.55	1.6	8/5
f	39.4	d/f	2.03	2.00	2/1

Some of these ratios are not exact. However the dimensions have already been used to support the Megalithic Yard theory, so they must by the same argument also support harmonic ratio theory.

In sites which use harmonic ratios, every single dimension makes a harmonic ratio with at least one other dimension. The probability that harmonic ratios on these sites are not a real phenomenon is very small.

Silbury Hill near Avebury was built as a 7 tiered circular chalk pyramid, which was then covered over with turf. Its design is 8 concentric circles.

Data from "The Silbury Treasure" by Michael Dames.
Once again "MY" means Megalithic Yards

Diameter of mound (Silbury 3 and 4)	192 MY	192/48 = 4/1
Height of mound (Silbury 3 and 4)	48 MY	144/48 = 3/1
Max diameter of ditch deep section (Silbury 2)	144 MY	144/88.5 = 8/5
Diameter of mound (Silbury 2)	88.8 MY	88.8/44.4 = 2/1
Diameter of mound (Silbury 1)	44.4 MY	192/144 = 4/3
Diameter of external summit flat	36.75 MY	36.5/24 = 3/2
Diameter of wattle fence (Silbury 1)	24 MY	24/6 = 4/1
Diameter of clay mound	6 MY	

There are many instances of fractions 8/5 and 5/3, which do not have the actual Egyptian sacred fraction forms (n+1)/n. However they are in the family of musical intervals. 8/5 is 8 steps around the circle of notes, and 5/3 is 9 steps.

The first six terms of the Fibonacci sequence, taken as ratios, are   1/1, 2/1, 3/2, 5/3, 8/5. These are the ratios that nature adopts under the influence of the sun's rays. Another site with sacred musical ratios is the Sanctuary at East Kennet.

3) SPHINX TEMPLE

Shown below is a very simplified sketched plan of the Sphinx temple.
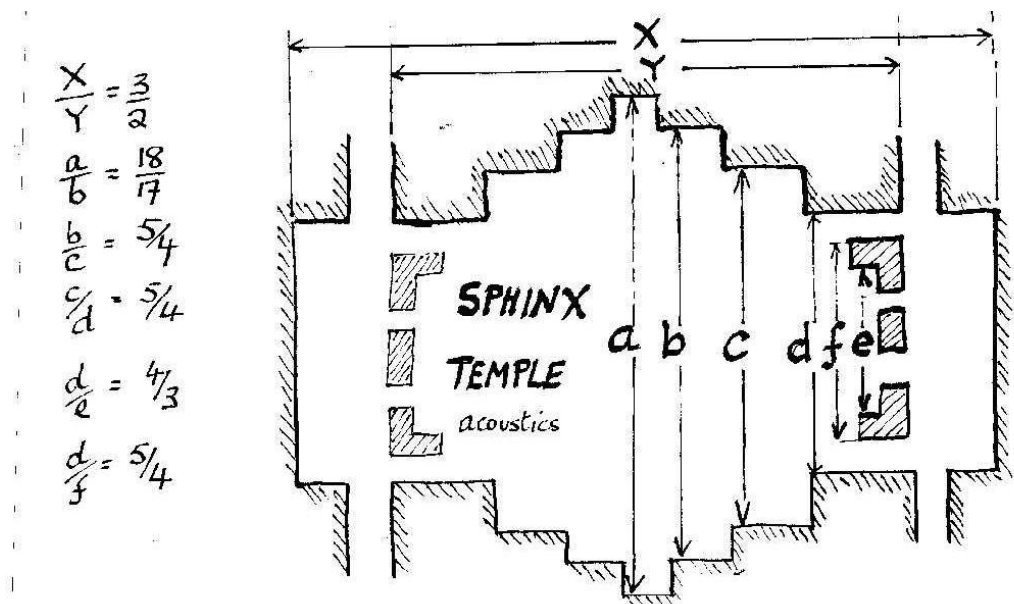
Temples were used for meditation. The priest would have a "directed dream" in which he would travel among the stars and visit gods "in trance". He needed music to carry him, and the acoustics of the temple were the essential feature of its design. The temple had flat stone walls of varying distances apart, which were all in whole number ratios with each other. The main shape, two squares, was for two octaves. Other dimensions form sacred musical ratios. Presumably these amplified the notes of the scale. The Egyptians used the same major scales and the "spiral of fifths" that we use today.
The proportions of the temple include the ratio 18/17, an exact semitone.

There are many more dimensions to be considered in this temple, and every one of them makes a harmonic ratio with at least one other dimension.

The temple was connected to the pyramid Khafre with water. Khafre is the pyramid with the 4/3 face slope that signifies travel in the Underworld. I believe that the purpose of the temple was a kind of "directed dreaming", and the dreamer imagined that he was in a boat, and was using the ratio 4/3 to propel this boat across the Underworld.

4) GILGAMESH

If Gilgamesh is read with sacred ratios in mind it becomes a version of 'Genesis' and Exodus. It was originally written in 2350 BC for King Sargon of Akkad, who claimed to be the reincarnation of Gilgamesh. It tells the story of his previous life and adventures as a god actually living on the earth.

This god was a "Prometheus" figure who brought souls down from heaven to give to humans. He was also doomed to become fully mortal and die. Sargon was saying that he was still in fact a god and is "immortal". After his death he would have to go down to the Underworld, but would be able to escape. He is also a proto-Orpheus figure.

The poem describes in detail how 2/3 was used to travel in trance, how Gilgamesh used it to visit the ancient "Noah" for advice, and how he killed Humbaba. Gilgamesh is "two thirds god". He _is_ the sacred fraction. The Epic refers to the "spiral of fifths" in several different ways (eg "like a twist of wool"). Time travel is inferred because Gilgamesh cannot put his hand in the water "lest it wither".

Four areas of investigation

Gilgamesh is a god who has become trapped in the world by the structure of music itself, "the cosmic laws". These laws are the sacred numbers (ratios) which are combined using simple maths to form equations.

Sacred numbers are interwoven with each of the four investigation areas. A change in one area always affects the three areas directly.

To understand what the Giza pyramids mean, or how the dimensions of the temple are important, you need to acknowledge the sacred ratios.

It is the science of sound. The ratios occur in quantum physics, they explain why energy exists in "quanta" (discrete quantities).

Musicians are taught that the laws are trivial and need not be bothered with. What "musician" can answer these questions?

Why are the notes of a major scale those particular 7 notes out of a possible 12?

Why are some chords "discordant"?

Surely these are elementary questions.

The way that the Egyptians and Pythagoras thought about mathematics, which could easily be taught in our schools, explains very neatly the answers to questions like these in concrete terms. It is visual tactile and not "abstract". It is practical and it might even be fun for modern children. It has moral musical and spiritual value.

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